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**MP 710: Advanced MRI**

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**QUIZ 2**

**Question 1 (2 points)**

Compressed sensing theory requires ‘incoherent sampling’ artefacts from undersampling.

a. How can that be achieved when using a (A) Cartesian 2D and a (B) Cartesian 3D acquisition?

For 2D Cartesian imaging (A), the only option is to undersample in the phase encode dimension since the full dataset is acquired in the frequency dimension (i.e., there would be no easy way to undersample in the frequency encode direction and save time).

For Cartesian 3D acquisitions, phase encodes are randomly undersampled in both the ky and kz domains. This would further promote incoherence and reducing incoherence.

b. Is one of these sampling schemes (Cartesian 2D vs Cartesian 3D) better suited to fulfill the requirement of incoherent sampling artefacts.

Cartesian 3D sampling is better suited to produce incoherent sampling artifacts because there is pseudo-randomness in *2 dimensions*. This would globally decrease the point spread function and also act to decrease the SPR.

**Question 2 (2 points)**

Besides sparsity, what are other conditions should be met for a compressed sensing reconstruction according to the CS theory?

- Aliasing artifacts due to k-space undersampling must be incoherent (noise-like) in the transform domain.

- Nonlinear reconstruction be used to enforce both sparsity of the image representation and consistency with the acquired data.

**Question 3 (2 points)**

Which of the following is a true statement about the Constrained Optimization problem used in CS image reconstruction?

(1) The L2 norm is used to measure the reconstructed image in the sparse transform domain since it penalizes large coefficients.

(2) The parameter ε represents the minimum allowable difference between the Fourier transformed reconstruction and the measured k‐space data (under the L2 norm).

(3) The sparse transform ψ may be any non‐linear operator besides the identity matrix.

(4) The use of the L1 norm in the sparse transform domain is crucial since it often results in a solution comprised of a small number of large coefficients.

**4.** Yes, this is how sparsity is promoted.

**Question 4 (4 points)**

Briefly explain how the sparse signal shown in Fig 2a can be properly recovered from pseudo‐random k-space undersampling shown in (b), resulting in the signal representation in (d) with a direct reconstruction. Hint: you don’t have to explore this with mathematical rigor, an intuitive description is sufficient.

Strong signal components in d), resulting from the (sparse) non-zero signal in image a), will stick out above the incoherent interference signal (similar to noise) caused by pseudo-random undersampling. From image d), we can set a threshold only signals above the interference level and transform the thresholded signal into a detected signal. However, this may not include some smaller signals in the imaging domain. If we calculate the interference from the detected signals (based on our known random sampling scheme), we can subtract this from the interference that was obtained from the full sample, resulting in a reduced interference signal that would be caused by other low signal points in the imaging domain. This can again be thresholded and transformed to pick out detected signals in the image. Doing this process iteratively was first described by David Donoho (Sparse Solution of Underdetermined Linear Equations by Stagewise, Orthogonal Matching Pursuit).